

Solutions - Homework 2

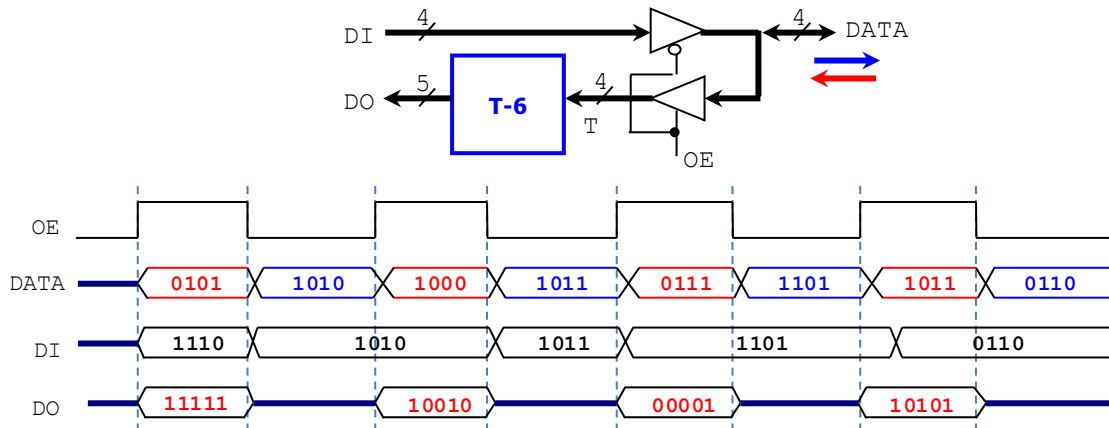
(Due date: October 7th @ 11:59 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (12 PTS)

- Complete the timing diagram (signals *DO* and *DATA*) of the following circuit. The circuit in the blue box computes the signed operation T-6, with the result having 5 bits. T is a 4-bit signed (2C) number.

For example: if $T=1010 \rightarrow DO = 1010 - 0110 = 11010 + 11010 = 10100$.



PROBLEM 2 (20 PTS)

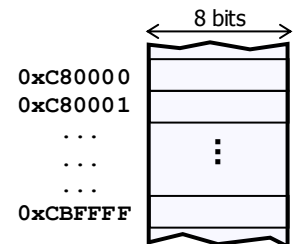
- a) What is the minimum number of bits required to represent: (2 pts)
- ✓ 220,000 symbols? $\lceil \log_2 220,000 \rceil = 18 \text{ bits}$
 - ✓ Numbers between (and including) 65,000 and 69,096? $\lceil \log_2 (69,096 - 65,000 + 1) \rceil = 13 \text{ bits}$

- b) A microprocessor has a 24-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (6 pts)

- What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes. (2 pts)

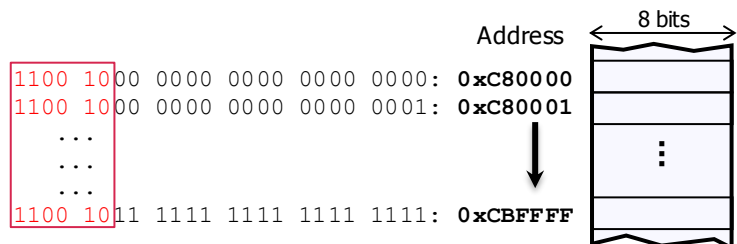
Address Range: $0x000000$ to $0xFFFFFFFF$

With 24 bits, we can address 2^{24} bytes, thus we have $2^{4 \times 20} = 16 \text{ MB}$



- A memory device is connected to the microprocessor. Based on the memory size, the microprocessor has assigned the addresses $0xC80000$ to $0xCBFFFF$ to this memory device.
 - What is the size (in bytes, KB, or MB) of this memory device?
 - What is the minimum number of bits required to represent the addresses only for this memory device?

As per the figure, we only need 18 bits for the addresses in the given range (where the memory device is located). Thus, the size of the memory device is $2^{18} = 256 \text{ KB}$.

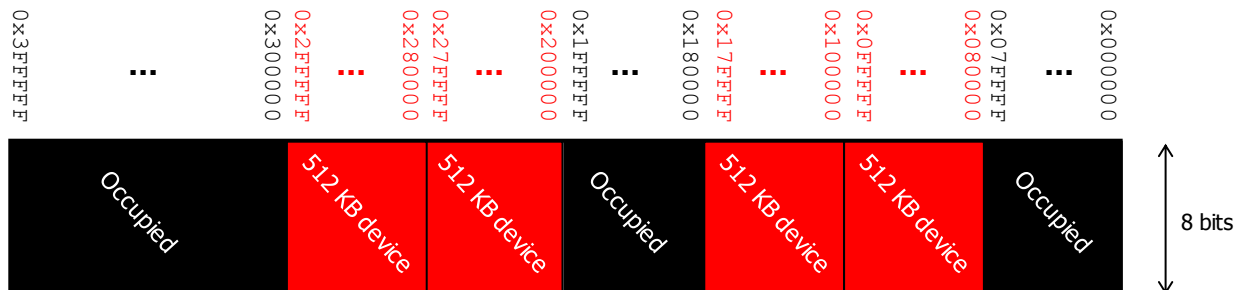


- c) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (12 pts)
- What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor? (2 pts)
- Address Range: $0x000000$ to $0xFFFFFFFF$. To represent all these addresses, we require 22 bits. So, the address bus size of the microprocessor is 22 bits. The size of the memory space is then $2^{22} = 4 \text{ MB}$.

- If we have a memory chip of 512KB, how many bits do we require to address 512KB of memory?
512KB memory device: $512\text{KB} = 2^{9210} = 2^{19}$ bytes. Thus, we require 19 bits to address the memory device.
- We want to connect the 512KB memory chip to the microprocessor. For optimal implementation, we must place those 512KB in an address range where every single address shares some MSBs (e.g.: 0×000000 to $0 \times 7FFFFF$). Provide a list of all the possible address ranges that the 512KB memory chip can occupy. You can only use the non-occupied portions of the memory space as shown below. (8 pts)

The 19-bit address range for an 512KB memory would be: 0×000000 to $0 \times 7FFFFF$. To place this range within the 22-bit memory space in the figure, we have four options:

0×080000 to $0 \times 0FFFFF$
 0×100000 to $0 \times 17FFFF$
 0×200000 to $0 \times 27FFFF$
 0×280000 to $0 \times 2FFFFF$



PROBLEM 3 (34 PTS)

- In ALL these problems (a, b, c, d), you MUST show your conversion procedure. **No procedure = zero points.**
 - Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (12 pts)
 - ✓ $-97.125, 63.3125, -64.65625, -71.25$.
 - $97.125 = 01100001.001 \rightarrow -97.125 = 10011110.111 = 0 \times 9E.E$
 - $63.3125 = 0111111.0101 = 0 \times 3F.5$
 - $64.65625 = 0100000.10101 \rightarrow -64.65625 = 1011111.01011 = 0 \times BF.58$
 - $71.25 = 01000111.01 \rightarrow -71.25 = 10111000.11 = 0 \times B8.C$
 - We want to represent integer numbers between (and including) -16384 to 16384 using the 2C representation. What is the minimum number of bits required? (2 pts)

Range of signed integer with n bits: $[-2^{n-1}, 2^{n-1} - 1]$

$$\Rightarrow 2^{n-1} - 1 \leq 16384 \rightarrow 2^{n-1} \leq 16385 \rightarrow n - 1 \geq \log_2 16385 \rightarrow n \geq 15.0000880524 \rightarrow n = 16$$

\therefore The minimum required number of bits is $n = 16$.

- Complete the following table. The decimal numbers are unsigned: (6 pts)

Decimal	BCD	Binary	Reflective Gray Code
269	001001101001	100001101	110001011
346	001101000110	101011010	111110111
418	010000011000	110100010	101110011
102	000100000010	1100110	1010101
110	000100010000	1101110	1011001
687	011010000111	101010111	111111000

- Complete the following table. Use the fewest number of bits in each case: (14 pts)

Decimal	REPRESENTATION		
	Sign-and-magnitude	1's complement	2's complement
-16	110000	101111	10000
-257	1100000001	1011111110	101111111
32	0100000	0100000	0100000
64	01000000	01000000	01000000
0	00	111111	0
-33	1100001	1011110	1011111
-31	1011111	100000	100001

PROBLEM 4 (34 PTS)

- a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (8 pts)

Example ($n=8$):

✓ $54 + 210$

$$\begin{array}{r}
 \begin{array}{c} c_8 \\ \text{No Overflow} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 54 = 0 \times 36 = & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & + \\
 210 = 0 \times D2 = & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & \\
 \hline
 \text{Overflow!} \rightarrow & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
 \end{array}$$

✓ $221 + 117$

✓ $76 + 175$

$$\begin{array}{r}
 \begin{array}{c} c_9 \\ \text{No Overflow} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 221 = 0 \times DD = & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & + \\
 117 = 0 \times 75 = & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & \\
 \hline
 \text{Overflow!} \rightarrow & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} c_9 \\ \text{No Overflow} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 175 = 0 \times AF = & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & + \\
 76 = 0 \times 4C = & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & \\
 \hline
 251 = 0 \times FB = & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 &
 \end{array}$$

✓ $77 - 194$

$$\begin{array}{r}
 \begin{array}{c} b_8 \\ \text{Borrow out!} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 77 = 0 \times 4D = & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & - \\
 194 = 0 \times C2 = & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & \\
 \hline
 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 &
 \end{array}$$

✓ $93 - 128$

✓ $130 - 43$

$$\begin{array}{r}
 \begin{array}{c} b_8 \\ \text{Borrow out!} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 93 = 0 \times 5D = & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & - \\
 128 = 0 \times 80 = & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 \hline
 0 \times DD = & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 &
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} b_8 \\ \text{No Borrow Out} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 130 = 0 \times 82 = & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & - \\
 43 = 0 \times 2B = & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & \\
 \hline
 87 = 0 \times 57 = & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 &
 \end{array}$$

- b) We need to perform the following operations, where numbers are represented in 2's complement (2C): (20 pts)

✓ $43 - 130$

✓ $156 + 359$

✓ $126 - 91$

✓ $87 - 62$

✓ $-127 - 66$

- For each case:

- ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
- ✓ Perform the signed (2C) binary addition. The result must have the same number of bits as the summands.
- ✓ Determine whether there is overflow by:
 - i. Using c_n, c_{n-1} (carries).
 - ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for n bits, where n is the minimum number of bits for the summands.
- ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

$n = 9$ bits

$c_9 \oplus c_8 = 0$
No Overflow

$$\begin{array}{r}
 \begin{array}{c} c_9 \\ \text{No Overflow} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 43 = 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & + \\
 -130 = 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & \\
 \hline
 -87 = 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & \\
 43 - 130 = -87 \in [-2^8, 2^8-1] \rightarrow \text{no overflow}
 \end{array}$$

$n = 8$ bits

$c_8 \oplus c_7 = 0$
No Overflow

$$\begin{array}{r}
 \begin{array}{c} c_8 \\ \text{No Overflow} \end{array} \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \\
 87 = 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & + \\
 -62 = 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & \\
 \hline
 25 = 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & \\
 87 - 62 = 25 \in [-2^7, 2^7-1] \rightarrow \text{no overflow}
 \end{array}$$

n = 10 bits

$c_{10} \oplus c_9 = 1$
Overflow!

$$\begin{array}{r} 156 = 000100111000 + \\ 359 = 0101100111 \\ \hline 1000000011 \end{array}$$

$156 + 359 = 515 \notin [-2^9, 2^9-1] \rightarrow \text{overflow!}$

To avoid overflow:

n = 11 bits (sign extension)

$c_{11} \oplus c_{10} = 0$
No Overflow

$$\begin{array}{r} 156 = 000100111000 + \\ 359 = 00101100111 \\ \hline 515 = 01000000011 \end{array}$$

$156 + 359 = 515 \in [-2^{10}, 2^{10}-1] \rightarrow \text{no overflow}$

n = 8 bits

$c_8 \oplus c_7 = 0$
No Overflow

$$\begin{array}{r} 126 = 01111110 + \\ -91 = 10100101 \\ \hline 35 = 00100011 \end{array}$$

$126 - 91 = 35 \in [-2^7, 2^7-1] \rightarrow \text{no overflow}$

n = 8 bits

$c_8 \oplus c_7 = 1$
Overflow!

$$\begin{array}{r} -127 = 10000001 + \\ -66 = 10111110 \\ \hline 00111111 \end{array}$$

$-127 - 66 = -193 \notin [-2^7, 2^7-1] \rightarrow \text{overflow!}$

To avoid overflow:

n = 10 bits (sign extension)

$c_9 \oplus c_8 = 0$
No Overflow

$$\begin{array}{r} -127 = 110000001 + \\ -66 = 10111110 \\ \hline -193 = 10011111 \end{array}$$

$-127 - 66 = -193 \in [-2^8, 2^8-1] \rightarrow \text{no overflow}$

c) Get the multiplication results of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)

✓ 0101×0101 , 1011×0111 , 1010×1110 .

$$\begin{array}{r} 0101 \times \\ 0101 \\ \hline 0101 \\ 0000 \\ 0101 \\ 0000 \\ \hline 00011001 \end{array}$$

$$\begin{array}{r} 1011 \times \\ 0111 \\ \hline 0101 \\ 0101 \\ 0101 \\ 0000 \\ \hline 00100011 \\ \downarrow \\ 11011101 \end{array}$$

$$\begin{array}{r} 1010 \times \\ 1110 \\ \hline 0110 \\ 0010 \\ 0110 \\ 0000 \\ \hline 00001100 \end{array}$$